Problem Selection from Past Spring 2025

Math Olympiad Club Zurich

Fall 2025

Problem B3 Putnam 2001

For each $n \in \mathbb{N}$, let $\langle n \rangle$ denote the closest integer to $\sqrt{n}$. Find $\sum_{n=1}^{\infty} \frac{2^{\langle n \rangle}+2^{-\langle n \rangle}}{2^n}$.

Problem (Hongler)

We have a linked list of elements, $x_0 \to x_1 \to \ldots \to x_n$, where $n \geq 1$ is unknown (but it is known that the list is finite). When at x_0, we have a pointer to go to x_1, which leads to x_2, and so on, until we reach x_n, where we learn that it is the end. We have a bounded amount of memory (at least we can stock one elements x_i) (we cannot simply store the entire list in an array and choose an element from the array once we reach the end).
(a) How can we select a random element in the linked list $x_1 \to \ldots \to x_n$, uniformly, if we are allowed to traverse the list only once?
(b) Why might solving this problem be useful in practice?

Problem 1 (Bernoulli Competition 2023)

1. Let $A = \{1, 2, \ldots, 100\}$ be the set of integers between 1 and 100.

(a) Let $B \subset A$ be a subset that doesn't contain two consecutive integers. What is the maximal cardinality of B?

(b) Let $C \subset A$ be a subset such that there is no n for which n and $2n$ are both in C. What is the maximal cardinality of C?