Problem Selection from Past Spring 2025

Math Olympiad Club Zurich

Spring 2025

Problem unknown

There is an odd number ($2 \cdot n + 1 > 0$) of stones with real weights satisfying the following property: if we remove any stone from the $2 \cdot n + 1$, then there is a way to partition the rest of the stones into two sets of size n, such that the sum of the weights of the stones in both sets is equal. Show that all stones have the same weight.

Problem B-6 IMC 2024

Show that any function $f : \mathbb{Q} \longrightarrow \mathbb{Z}$ satisfy the following propertie:

$$\exists a, b, c \in \mathbb{Q} \text{ with } a < b < c \text{ such that } f(a), f(c) \leq f(b)$$

Problem (Selected Real Analysis Problem)

For each function $g \in \{-id_{\mathbb{R}}, \exp, x \mapsto x^2 - 2\}$, determine wheter there exists a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f \circ f = g$.

Bonus: Solve the same problem with $g \in \{\cos, \sin\}$.

Problem B4 (Putnam 2001)

Let $S := \mathbb{Q} \setminus \{-1, 0 - 1\}$. Define $f : S \to S$ by $f(x) = x - \frac{1}{x}$. Prove or disprove that

$$\bigcap_{n=1}^{\infty} f^{(n)}(S) = \emptyset,$$

where $f^{(n)}$ denotes f composed with itself n times.