Problem Set Week 4

ETHZ Math Olympiad Club

17 March 2025

Problem (unknown)

Find all real solutions to the equation

$$9^x + 4^x + 2^x = 8^x + 6^x + 1.$$

Problem 2 (Bernoulli Competition 2023)

Let e be Euler's number. Show that for any odd prime p, the integer

$$1! + 2! + 3! + \cdots + (p-1)! - \left\lfloor \frac{(p-1)!}{e} \right\rfloor$$

is divisible by p.

Problem in example page 140 (PUTNAM and BEYOND)

Find all real solutions to the equation

$$4^x + 6^{x^2} = 5^x + 5^{x^2}.$$

Problem 3 (Bernoulli Competition 2023)

Let $n \geq 1$ and A be a $n \times n$ symmetric matrix over $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$ with $1_{\mathbb{F}_2}$'s on the main diagonal. Show that the vector composed uniquely of $1_{\mathbb{F}_2}$'s is in the image of A.

Problem (unknown)

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Find all differentiable functions $f : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ having at least one fixed point $\alpha \in \mathbb{R}_{>0}$ satisfying:

$$f' = \frac{f}{f \circ f}.$$

Bonus: What happens if f has no fixed point?